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**LARGE TREE LEVEL CP VIOLATION IN $e^+e^- \rightarrow t\bar{t}H^0$ IN THE
TWO-HIGGS DOUBLET-MODEL**

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Abstract

We find a large CP violation effect within the Two-Higgs-Doublet-Model for the reaction $e^+e^- \rightarrow t\bar{t}H^0$ at future linear colliders. The CP-asymmetry arises already at the tree level as a result of interference between diagrams with H^0 emission from t (and \bar{t}) and its emission from a Z^0 and can be about 10–20%. In the best case one needs a few hundred $t\bar{t}H^0$ events to observe CP violation at the 3σ level.

Future high energy e^+e^- colliders ($500 \leq E_{cm} \leq 1000$ GeV) will serve as a very useful laboratory for the study of Higgs and top physics beyond the Standard Model (SM) [1]. The top quark with a mass of 176 GeV [2], being so heavy, is likely to be sensitive to the short distance physics underlying the SM above the electroweak scale. Searches for CP-violation in top physics should be a particularly useful probe of physics beyond the SM, since it is unlikely that the CP violating KM phase [3] in the SM can account for the observed baryon asymmetry in the universe [4]. One of the simplest extensions of the SM is the Two-Higgs-Doublet-Model (THDM) where one of the two doublets is responsible for giving masses to the charge $+2/3$ quarks and the other to the charge $-1/3$ quarks. This is also the preferred supersymmetry-motivated THDM [5, 6]. We recall that CP violation in top quark physics in such THDM has received considerable attention in the past few years [7].

In this Letter we focus on CP violation, driven by THDM in the process $e^+e^- \rightarrow t\bar{t}H^0$ at future e^+e^- colliders, where H^0 is the lightest neutral Higgs in the THDM. Within the SM, Higgs production in e^+e^- colliders was studied earlier at low [8] and high [9] energies. It was also studied in the context of general THDMs, in Z decays [10], where recently CP violation in Higgs production within the THDM was examined in [11]. We should emphasize that the reaction we study is not meant (necessarily) to lead to the discovery of H^0 but rather to investigate the CP-properties of H^0 . Clearly even after the H^0 is discovered its role in CP violation will need to be understood. This issue will thus be the main subject of our investigation.

A very interesting feature of the reaction $e^+e^- \rightarrow t\bar{t}H^0$ is that it exhibits a CP asymmetry at the tree graph level. Such an effect arises from interference of the Higgs emission from t or \bar{t} with the Higgs emission from the Z boson. Being a tree level effect the resulting asymmetry is quite large. This asymmetry can be detected through a CP-odd T_N -odd observable (T_N is the naive time reversal operator defined

by replacing time with its negative without switching initial and final states.) In the best scenario one needs a few hundred $t\bar{t}H^0$ events to observe CP violation at the 3σ level.

In the THDM CP-violation may emanate from the neutral Higgs sector. In general, the manifestation of such CP-violation is that the neutral Higgs mass eigenstates couple to fermions with both scalar and pseudoscalar couplings.

For $e^+e^- \rightarrow t\bar{t}H^0$ the following interaction terms in \mathcal{L} are required [5]:

$$\mathcal{L}_{H_j^0} = H_j^0 \bar{f} (a_{fj} + i b_{fj} \gamma_5) f + H_j^0 c_j g_{\mu\nu} Z^\mu Z^\nu + \frac{c_j}{2M_Z} [\chi^0 (\partial_\mu H_j^0) - (\partial_\mu \chi^0) H_j^0] Z^\mu , \quad (1)$$

which involves the $f\bar{f}H_j^0$, ZZH_j^0 and $Z\chi^0H_j^0$ couplings. Here f stands for a fermion, χ^0 is the unphysical Goldstone boson and H_j^0 is a neutral Higgs species. The three coupling constants, a_{fj} , b_{fj} and c_j are functions of $\tan\beta$, which is the ratio between the two vacuum expectation values in this model, i.e. $\tan\beta = v_2/v_1$, and of the three mixing angles, α_1 , α_2 and α_3 which diagonalize the Higgs mass matrix [5]. In particular

$$\begin{aligned} a_{fj} &= -2^{\frac{1}{4}} G_F^{\frac{1}{2}} m_f R_{2j} / \sin\beta , \quad b_{fj} = -2^{\frac{1}{4}} G_F^{\frac{1}{2}} m_f R_{3j} \cot\beta , \\ c_j &= 2^{\frac{5}{4}} G_F^{\frac{1}{2}} M_Z^2 (R_{j1} \cos\beta + R_{j2} \sin\beta) . \end{aligned} \quad (2)$$

R is the rotation matrix given by:

$$R = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 & c_1 c_2 s_3 + s_2 c_3 \\ s_1 s_2 & -c_1 s_2 c_3 - c_2 s_3 & -c_1 s_2 s_3 + c_2 c_3 \end{pmatrix} , \quad (3)$$

where $s_i \equiv \sin \alpha_i$ and $c_i \equiv \cos \alpha_i$.

We now discuss the tree-level cross-section and CP-violation effects in our reaction,

$$e^+(p_+) + e^-(p_-) \rightarrow t(p_t) + \bar{t}(p_{\bar{t}}) + H^0(p_H) . \quad (4)$$

We assume that two of the three neutral Higgs particles are much heavier than the remaining one, i.e. H^0 . We therefore omit the index j in Eqs. 1 and 2, and denote the couplings as: a_t , b_t and c . An important property of this simple reaction, is that it gives rise to CP-violation already at tree-level, as a result of interference of the diagram with H^0 emitted from the Z with the diagram where H^0 is radiated off the t or \bar{t} . The tree-level differential cross section Σ^0 is a sum of two terms: the CP-even and odd terms Σ_+^0 and Σ_-^0 , respectively, i.e. $\Sigma^0 \equiv \Sigma_+^0 + \Sigma_-^0$. Σ_{\pm}^0 are calculated from the tree-level diagrams in Fig. 1.

The incoming left or right polarized electron-positron current can be written as:

$$J_e^{\mu(j)} = \bar{v}_e(p_+) \gamma^\mu \frac{1 + j\gamma_5}{2} u_e(p_-) , \quad (5)$$

where $j = -1(1)$ for left(right) handed electrons. We write the tree-level amplitude as:

$$\mathcal{M}^0 = \sum_{\alpha} \sum_{\rho} \mu_{\rho}^{\alpha} , \quad (6)$$

where ρ indicates the diagram ($\rho = i, ii, iii$ for diagrams i, ii, iii , respectively in Fig. 1) and α indicates the gauge particle exchanged in the s -channel, i.e. $\alpha = Z, \gamma$. We then write the general form of μ_{ρ}^{α} as:

$$\mu_\rho^\alpha = J_e^{\mu(j)} \bar{u}(p_t) H_{\rho\mu}^\alpha v(p_{\bar{t}}) , \quad (7)$$

where $H_{\rho\mu}^\alpha$, corresponding to each diagram, are given by:

$$H_{i\mu}^Z = -C_Z \pi_t (a_t + i b_t \gamma_5) (\not{p}_t + \not{p}_H + m_t) \gamma_\mu C_{LR}^+ , \quad (8)$$

$$H_{i\mu}^\gamma = H_{i\mu}^Z (C_Z \rightarrow -C_\gamma, C_{LR}^+ \rightarrow 1) , \quad (9)$$

$$H_{ii\mu}^Z = C_Z \pi_{\bar{t}} \gamma_\mu C_{LR}^+ (\not{p}_{\bar{t}} + \not{p}_H - m_t) (a_t + i b_t \gamma_5) , \quad (10)$$

$$H_{ii\mu}^\gamma = H_{ii\mu}^Z (C_Z \rightarrow -C_\gamma, C_{LR}^+ \rightarrow 1) , \quad (11)$$

$$H_{iiii\mu}^{ZZ} = C_Z \pi_{ZH} c \gamma_\mu C_{LR}^+ , \quad (12)$$

$$H_{iiii\mu}^{Z\chi^0} = C_Z \pi_{ZH} (c_R^t - c_L^t) m_t \gamma_5 p_{H^0\mu} / m_Z^2 , \quad (13)$$

where $C_Z \equiv \pi \alpha \pi_z c_j^e / c_W^2 s_W^2$, $C_\gamma \equiv 4 \pi \alpha Q_q \pi_\gamma$. Q_q is the charge of the quark in the final state and $c_W(s_W)$ stands for $\cos \theta_W (\sin \theta_W)$. $c_j^e = c_L^e (c_R^e)$ for $j = -1(1)$ where $c_L^f = -2I_3^f + 2Q_f s_W^2$ and $c_R^f = 2Q_f s_W^2$. π_Z and π_γ are the Z boson and photon propagators, respectively and:

$$\begin{aligned} \pi_t &\equiv \frac{1}{2p_t \cdot p_{H^0} + m_{H^0}^2} , \quad \pi_{\bar{t}} \equiv \frac{1}{2p_{\bar{t}} \cdot p_{H^0} + m_{H^0}^2} \\ \pi_{ZH} &\equiv \frac{1}{P^2 - 2P \cdot p_{H^0} + m_{H^0}^2 - m_Z^2} . \end{aligned} \quad (14)$$

Furthermore, $P \equiv p_- + p_+$ and $C_{LR}^+ \equiv c_L^t L + c_R^t R$, $L, R = (1 \mp \gamma_5)/2$. With the above definitions Σ^0 is given by:

$$\Sigma^0 = \frac{1}{2} \sum_j | \sum_\alpha \sum_\rho \mu_\rho^\alpha |^2 , \quad (15)$$

where the sum over j is carried over the polarization of e^+ , t and \bar{t} . Also $\rho = \{i, ii, iii\}$ and $\alpha = Z, \gamma$. The factor of $\frac{1}{2}$ is due to the fact that we consider an unpolarized e^+ beam colliding with a polarized e^- beam. The expression for Σ_+^0 is quite involved and will not be given explicitly here.

For illustration, we adopt the value $\tan\beta = 0.5$ which gives large effects and is allowed by present experiments for $m_{H^+} \geq \mathcal{O}(400 \text{ GeV})$, where H^+ is the charged Higgs boson of the THDM [12, 13]. We plot the tree-level cross-section for $e^+e^- \rightarrow t\bar{t}H^0$ in Fig. 2 for $m_{H^0} = 100$ and 160 GeV , for two possible sets of the Higgs coupling constants a_t , b_t and c . Set I corresponds to $\tan\beta = 0.5$, $\alpha_1 = \pi/4$, $\alpha_2 = \pi/4$, $\alpha_3 = 0$ and set II - to: $\tan\beta = 0.5$, $\alpha_1 = \pi/4$, $\alpha_2 = \pi/2$, $\alpha_3 = 0$. The CP-violating piece of the tree-level differential cross-section is:

$$\begin{aligned} \Sigma_-^0 &= 2C_Z \frac{m_t}{M_Z^2} \pi_{ZH} c b_t \times E \times \{ j \times (\pi_t + \pi_{\bar{t}}) \\ &\quad \times \left[(s - s_t - M_H^2)(C_Z(c_R^t + c_L^t) - 2C_\gamma) - 4C_\gamma(c_R^t - c_L^t)M_Z^2 \right] \\ &\quad + 2C_Z f(c_R^t - c_L^t)(\pi_t - \pi_{\bar{t}}) \} , \end{aligned} \quad (16)$$

where: $E \equiv \epsilon(p_-, p_+, p_t, p_{\bar{t}})$, $s \equiv 2p_- \cdot p_+$, $s_t \equiv (p_t + p_{\bar{t}})^2$, $f \equiv (p_- - p_+) \cdot (p_t + p_{\bar{t}})$ and $j = -1(1)$ for a left(right) handed electron.

Of course, at tree-level there are no absorptive phases. Thus the CP-violating term Σ_-^0 can probe only CP-asymmetries of the T_N -odd type. This leads us to consider the following CP-odd, T_N -odd, triple correlation product

$$O = \vec{p}_- \cdot (\vec{p}_t \times \vec{p}_{\bar{t}})/s^{3/2} . \quad (17)$$

To observe a non-vanishing average value $\langle O \rangle$ with a statistical significance of σ in an ideal experiment, one needs:

$$N_{t\bar{t}H^0}^\sigma = \sigma^2/A_O^2 , \quad (18)$$

events, where A_O is given by:

$$A_O \equiv \langle O \rangle / \sqrt{\langle O^2 \rangle} . \quad (19)$$

The number of expected $t\bar{t}H^0$ events is $N_{t\bar{t}H^0} = \mathcal{L} \times \sigma(e^+e^- \rightarrow t\bar{t}H^0)$ where \mathcal{L} is the collider luminosity. Fig. 3 shows our main results for $m_H = 100$ and 160 GeV, for set II of $\tan\beta, \alpha_1, \alpha_2$ and α_3 . We have also used $m_t = 176$ GeV [2] and took the electron to be unpolarized. We have depicted the number of events, N_O , required to observe a non-vanishing value (to one sigma) for the T_N -odd observable $\langle O \rangle$. We have also plotted in Fig. 3 the expected number of $t\bar{t}H^0$ events per year, N_{exp} , in an e^+e^- linear collider with a luminosity of $\mathcal{L} = 3 \times 10^{33} \text{cm}^{-2}\text{sec}^{-1}$ for CM energies of $\sqrt{s} \cong 500 - 1000$ GeV. We see that near threshold, at $\sqrt{s} \cong 500$ GeV, CP-violation asymmetry is far too small to be observed, i.e. $N_O \gg N_{\text{exp}}$. However, in this scenario N_O and N_{exp} do cross each other. For $m_{H^0} = 100$ GeV the crossing appears at $\sqrt{s} \cong 800$ GeV and for $m_{H^0} = 160$ GeV at $\sqrt{s} \cong 850$ GeV. This crossing means that for set II of the parameters, one may be able to observe CP-violation (to one sigma) in the process $e^+e^- \rightarrow t\bar{t}H^0$, at CM energies of $\sqrt{s} \cong 800 - 1000$ GeV and for Higgs masses of $100 - 160$ GeV. For example, for $m_{H^0} = 100$ GeV and at $\sqrt{s} \cong 1000$ GeV, $N_O/N_{\text{exp}} \cong 0.65$. The results for the set I of parameters do not look as promising. For example, for $\sqrt{s} \cong 1000$ GeV and $m_{H^0} = 100$ or 160 GeV we obtain $N_O/N_{\text{exp}} \cong 4$. Typically, at least several hundreds of events are needed in this case for a 1σ effect, as opposed to tens of events for set II.

In Fig. 4 we show the dependence of the ratio N_O/N_{exp} on $\tan\beta$ for $m_H = 100$ and 160 GeV and for $\sqrt{s} = 800$ and 1000 GeV. We have kept all other parameters the same as in Fig. 3. We see that N_O/N_{exp} depends only mildly on $\tan\beta$ for $0.2 \lesssim \tan\beta \lesssim 1$.

For a given model of CP-violation it can be shown [14] that the optimal observable to use is given by:

$$O_{\text{iopt}} = \frac{\Sigma_-^{Im}}{\Sigma_+^0} , \quad O_{\text{ropt}} = \frac{\Sigma_-^{Re}}{\Sigma_+^0} , \quad (20)$$

where the superscripts Im and Re refer to that part of the amplitude proportional to the sin or cos of an absorptive phase. Since absorptive effects require at least 1-loop, the T_N -even asymmetry such as for O_{iopt} are smaller by a factor of order α_s/π .

In Table 1 we present our results (for set II) for the number of events required to detect a non-vanishing $\langle O \rangle$ and $\langle O_{\text{ropt}} \rangle$ (to one sigma) for 3 different CM energies, with $m_{H^0} = 100$ or 160 GeV. To illustrate the effect of polarization, we have included in the table results for O_{ropt} and O for different polarizations of the electron. Of course, O_{ropt} is related to O by multiplication by a CP-even function since there is only one possible independent triple product correlation when the final-state consists of three particles only. As can be seen from the numbers, the CP asymmetries (see eqns. 18 and 19), are in the 10–20% range and O gives almost as good results as O_{ropt} .

Note that to be able to measure the above observables, one would have to reconstruct the transverse components of the t and the \bar{t} momenta in each $t\bar{t}H^0$ event. This may be difficult in practice. Therefore, we present results for an additional

observable which requires only the determination of the momenta of the b and \bar{b} in the process $e^+e^- \rightarrow t\bar{t}H^0 \rightarrow bW^+\bar{b}W^-H^0$. Define the observable:

$$O_b = \epsilon(p_-, p_+, p_b, p_{\bar{b}})/s^2 \quad (21)$$

In Table 2 we present our results for O_b . It is evident that the number of events required to observe a 1σ CP-violating effect is comparable to the numbers found with the observable O , in particular, for $\sqrt{s} \cong 800 - 1000$ GeV. Close to threshold, e.g. $\sqrt{s} \cong 600$ GeV, the effect would be much harder to observe through O_b .

There are two other comments that we wish to make in brief. First we note that the $t\bar{t}H^0$ final state is expected to be the focus of intense scrutiny to unravel in detail the interaction of the Higgs with the top quark. Thus it is especially gratifying that the promising signal for CP violation that our study indicates are expected in the same final state. We note also that the method seems most suitable for a Higgs of mass $\lesssim 160$ GeV. Such a Higgs will decay predominantly into $b\bar{b}$ with a BR of $O(1)$.

To summarize, CP-violation in Higgs emission at a future high energy e^+e^- collider was investigated within the THDM. In the SM such a CP asymmetry will vanish at least to two loop orders in perturbation theory and therefore is expected to be extremely small. In contrast, in the THDM, an important and very interesting property of the reaction $e^+e^- \rightarrow t\bar{t}H^0$ is that the CP-violation arises already at tree-level through interference of H^0 emission from t or \bar{t} and its emission off a Z -boson, and therefore allows for large CP-violation effects. It is clearly important to examine this effect in other extensions of the SM. Our main result is that the CP-asymmetry in this process could be observable at a future linear e^+e^- collider with a luminosity of the order $\mathcal{L} \approx 10^{33} \text{ cm}^{-2}\text{sec}^{-1}$ running at CM energies around 800-1000 GeV.

We also showed that the reaction is quite promising for $0.2 \lesssim \tan \beta \lesssim 1$ although $\tan \beta \simeq 0.5$ seems to be the most suitable.

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Figure Captions

Fig. 1: Tree-level Feynman diagrams contributing to $e^+e^- \rightarrow t\bar{t}H^0$ within the two Higgs doublet model.

Fig. 2: The cross section for the reaction $e^+e^- \rightarrow t\bar{t}H^0$, for sets I and II of the parameters a_t , b_t and c and for $m_{H^0} = 100$ and 160 GeV assuming unpolarized electron and positron beams.

Fig. 3: Number of events, N_O , required to detect CP-violation via $\langle O \rangle$ at 1σ level and the expected yearly number of events N_{exp} , as a function of total beam energy for set II of the parameters and for $m_{H^0} = 100$ and 160 GeV with unpolarized electron and positron beams.

Fig. 4: N_O/N_{exp} versus $\tan\beta$ for $m_{H^0} = 100$ and 160 GeV and $\sqrt{s} = 800$ and 1000 GeV. The other parameters are held the same as in the previous figures.

Table 1: The number of events needed to detect $\langle O \rangle$ and $\langle O_{\text{ropt}} \rangle$ at 1σ is given for sets II of the parameters a_t , b_t and c . The left and right polarization ($j = -1$ and 1 , respectively) is compared with the unpolarized case. The values of \sqrt{s} and m_{H^0} are given in GeV.

\sqrt{s}	j	Set II			
		O		O_{ropt}	
		$m_{H^0} = 100$	$m_{H^0} = 160$	$m_{H^0} = 100$	$m_{H^0} = 160$
600	-1	100	70	95	65
	unpol	85	55	80	55
	1	60	40	60	40
800	-1	60	40	50	40
	unpol	50	35	45	35
	1	40	25	35	25
1000	-1	40	35	35	30
	unpol	40	30	30	25
	1	30	25	25	20

Table 2: The same as table 2 except for $\langle O_b \rangle$.

\sqrt{s}	j	Set II	
		O_b	
		$m_{H^0} = 100$	$m_{H^0} = 160$
600	-1	185	180
	unpol	205	270
	1	275	1280
800	-1	70	50
	unpol	65	45
	1	55	40
1000	-1	50	35
	unpol	45	30
	1	35	25

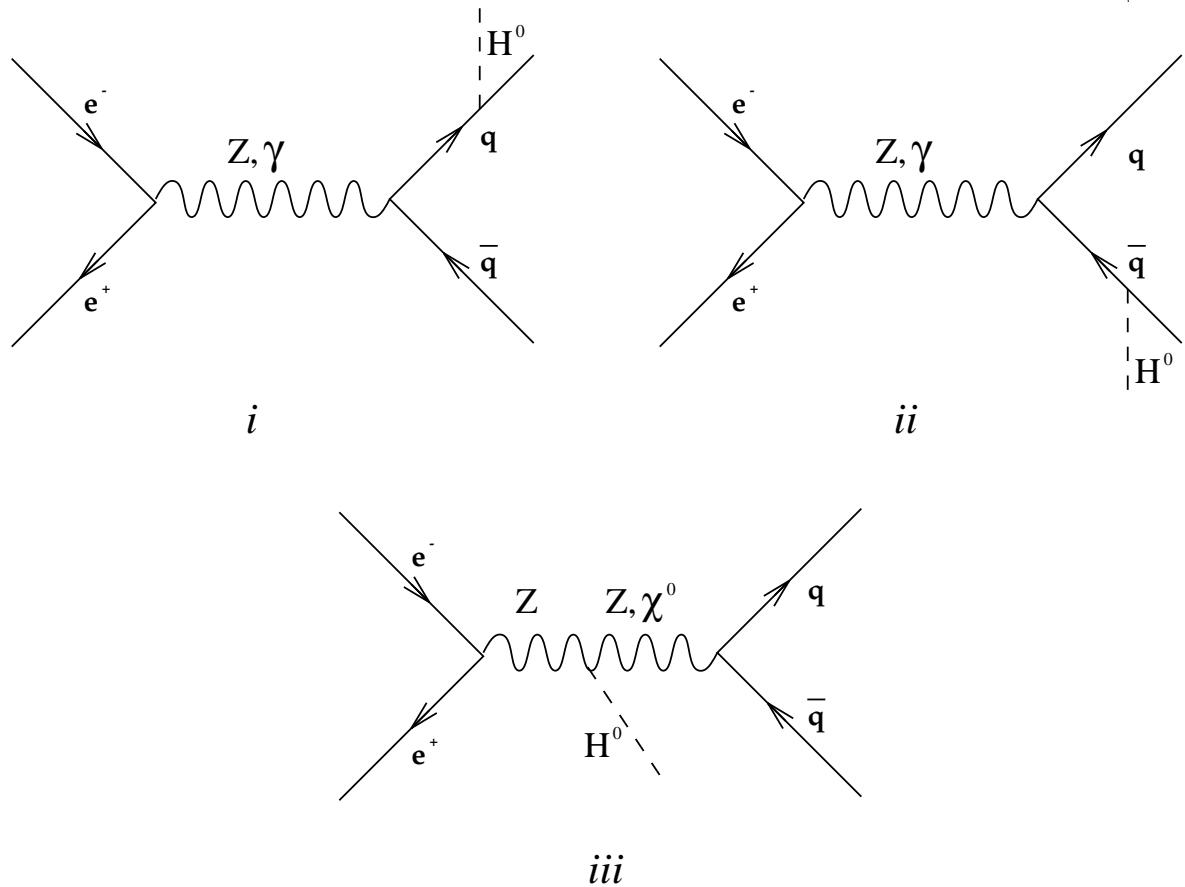


Fig. 1

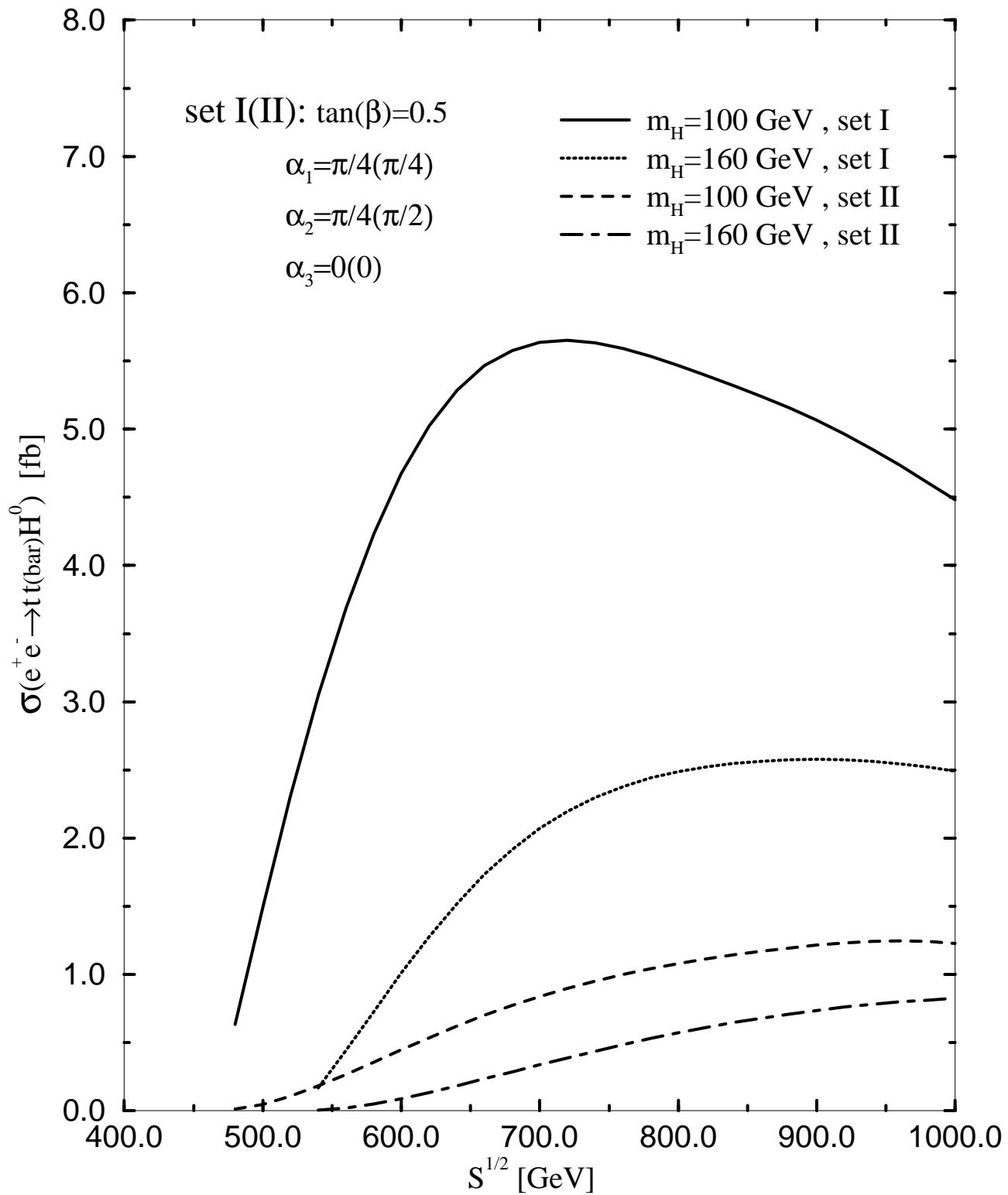


Fig.2

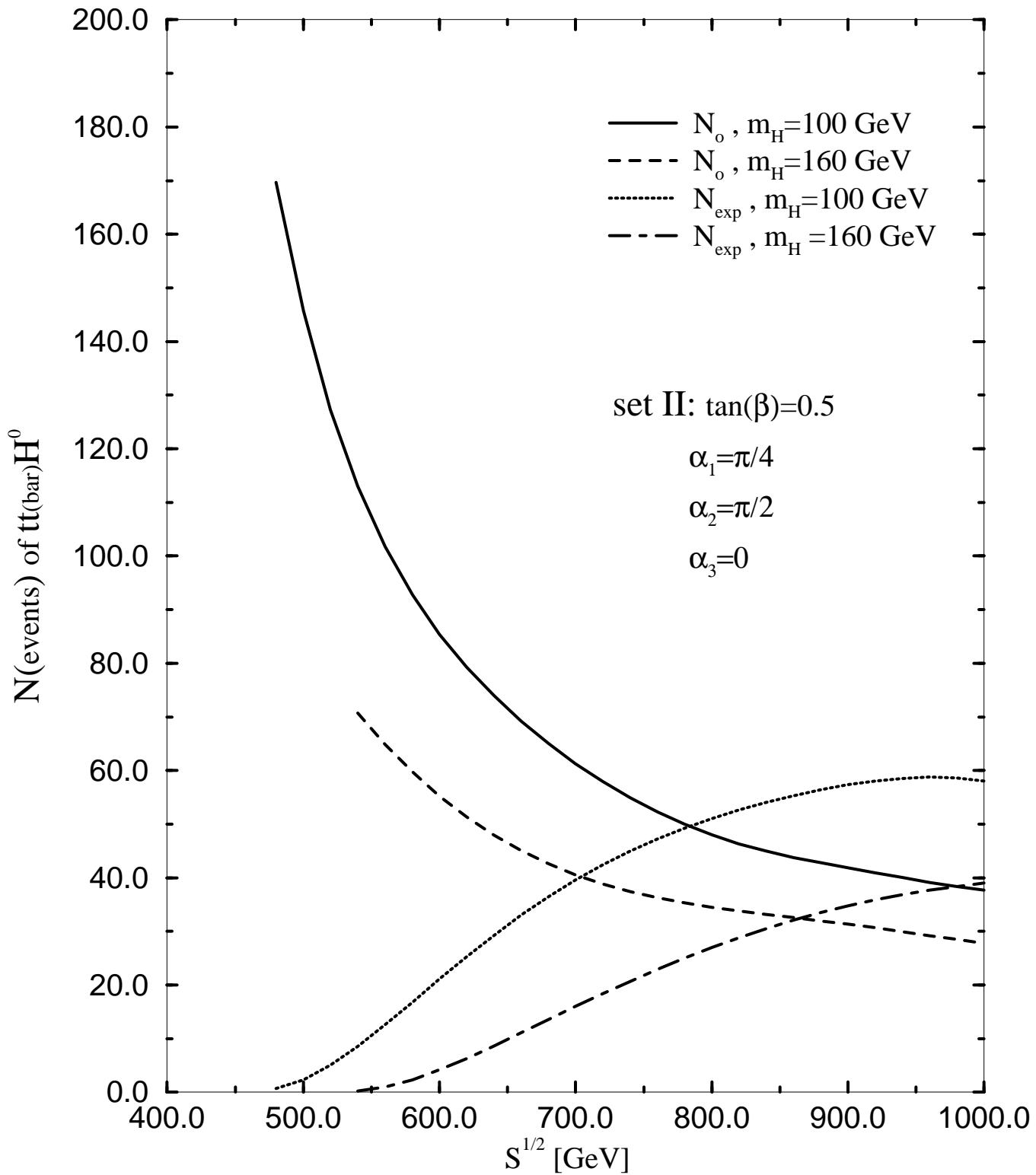


Fig.3

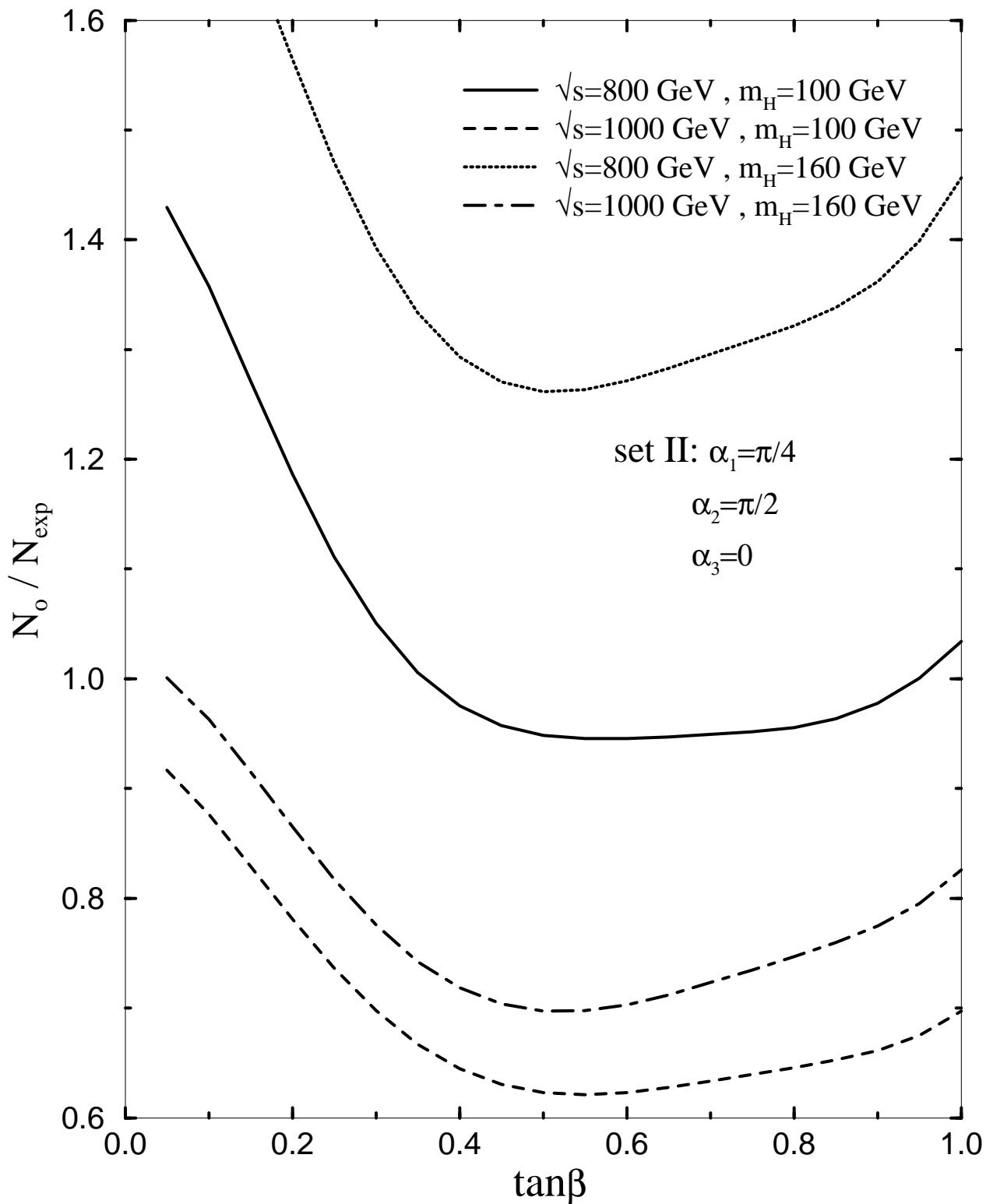


Fig.4